MATHEMATICAL MODEL FOR SINGLE-PASS CROSSFLOW HEAT EXCHANGER

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1. Introduction

Basic idea of the heat exchanger is to transfer heat from the hot fluid flow to the cold one. This phenomenon can be done with many various equipment and in this report it is presented how to evaluate heat recovery process in the plate type heat echanger where heat is recovered from the hot air flow to the cold air. Flowsheet and key factors for this type of crossflow heat exchanger is presented in figure 1.

Physically plate exchanger is a set of metal plates, which separate hot and cold fluid flows from each other. General layout of the heat exchangers is presented in figure 2.

The basic system is the heat exchanger where all fluid flows remain in gas phase and no condensing occurs over the heat surfaces. This type of system can be defined with the basic energy equations. The special case is presented when the temperature of the heat surface decreases below the saturation temperature (dew-point) of the humid air. In this situation the vaporised water will condense and release its latent energy. To model this exchange process we need simultaneous modelling of energy and mass transfer.

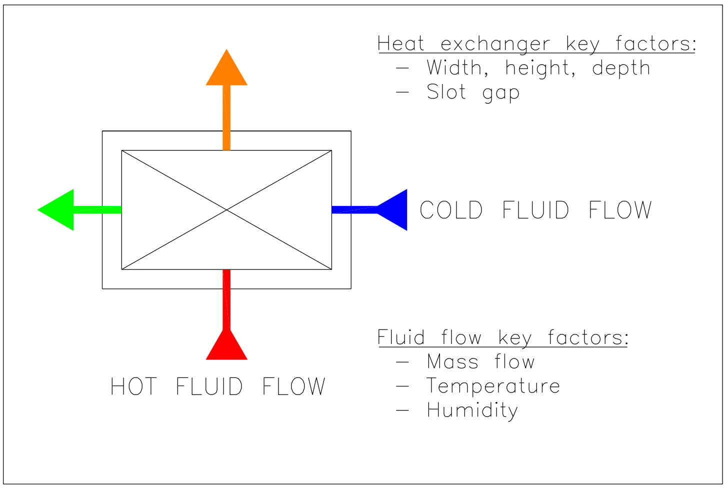

FIGURE 1. Flow sheet for the crossflow heat exchanger

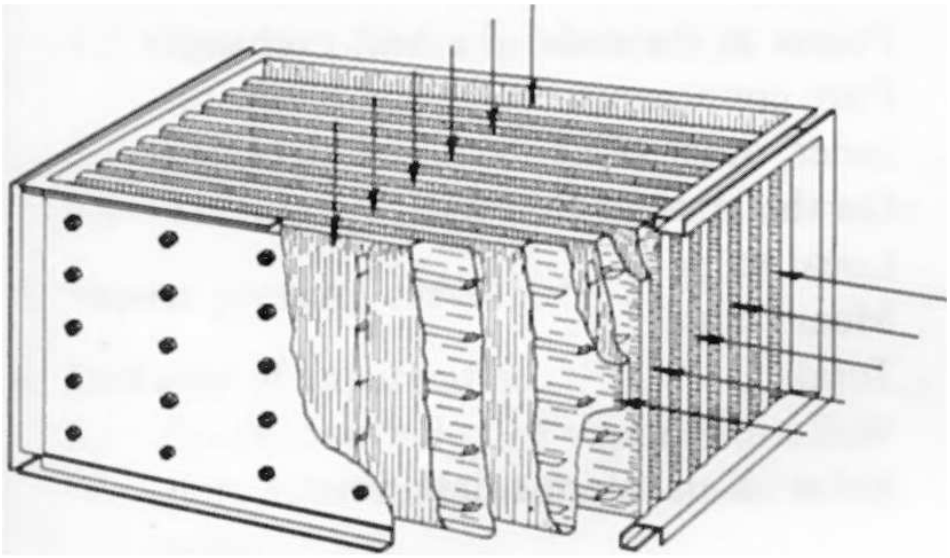

FIGURE 2. Physical layout of the crossflow heat exchanger

2. NTU-method Solution for Pure Crossflow Heat Exchanger Without Phase Change

The NTU-method (**n**umber of **t**ransfer **u**nits) is one of the traditional solution methods for the heat exchanger problems. The main limitation of this method is that it is difficult to handle heat exchangers where heat transfer coefficient changes remarkably over the heat surface. However in this report the main target is to develop a sufficient model for dry surface heat

exchanger and NTU-solution is to be used as a reference point for the new solution method. The equations and routines for NTU-method are widely presented in many references like [Incropera 1990] and [VDI-Heat Atlas, 1993] and method is described here only generally.
The general assumptions for the problem solution are as follows:

(1) All physical properties for incoming fluid flows are known.
(2) Heat exchanger is insulated from its surroundings, in which case the only heat exchange is between hot and cold fluids.
(3) Conduction along the walls is negligible.
(4) Potential and kinetic energy changes are negligible.
(5) Fluid is flowing only in one direction inside each heat exchanger tube or channel.
(6) All fluids enter the heat exchanger with a uniform velocity.
(7) Overall heat transfer coefficient changes over the heat surfaces are negligible.

When there is no condensation or evaporation on the heat surfaces the total heat transfer rate of the cross-flow heat exchanger can be calculated with the equation

$$\Phi = \mu f \Delta T_{ln} A_{tot}, \tag{1}$$

where

Φ	heat transfer rate	[W]
f	correction factor to fix the error got from the assumption for the log-mean-temperature	[-]
μ	overall heat transfer coefficient	[W/m^{2}K]
ΔT_{ln}	log mean temperature difference computed under the assumption of counter-flow conditions	[K]
A_{tot}	Total effective heat surface area	[m^2]

Log mean temperature difference can be calculated

$$\begin{aligned} \Delta T_{ln} &= \frac{\Delta T_1 - \Delta T_2}{\ln \frac{\Delta T_1}{\Delta T_2}} \\ \Delta T_1 &= T_{\text{in}} - \tilde{T}_{\text{out}} \\ \Delta T_2 &= T_{\text{out}} - \tilde{T}_{\text{in}} \end{aligned} \tag{2}$$

The overall heat transfer coefficient for the heat exchanger is defined:

$$\mu = \frac{1}{\frac{1}{h_{cold}} + \frac{s}{\lambda} + \frac{1}{h_{hot}}} \tag{3}$$

where

h_{cold}	convection coefficient between cold fluid flow and heat surface	[W/m^{2}K]
h_{hot}	convection coefficient between hot fluid flow and heat surface	[W/m^{2}K]
s	thickness of the heat exhanger wall	[m]
λ	conductivity of the heat exhanger wall	[W/mK]

The correction factor f can be either checked from the dimensioning diagrams for the pure cross-flow heat exchanger (See figure 3) or to be calculated with the equations presented in [VDI Heat Atlas, p. Ca 7-9].

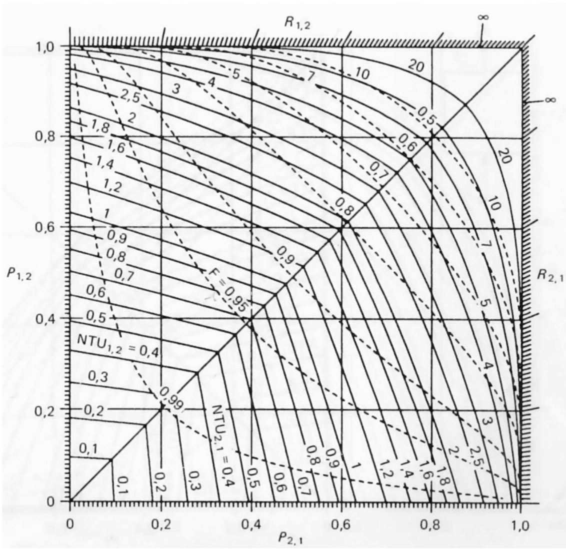

FIGURE 3. Dimensioning diagram for the pure crossflow heat exchanger [VDI Heat Atlas, p. Ca 25]

The dimensioning base for the sample system is:

$$
\begin{aligned}
q_{m,cold} &= 24,3 \quad \text{kg/s} \\
T_{cold,in} &= 30 \quad {}^{o}\text{C} \\
x_{cold} &= 20 \quad \text{g/kg} \\
q_{m,hot} &= 28,0 \quad \text{kg/s} \\
T_{cold,in} &= 85 \quad {}^{o}\text{C} \\
x_{cold} &= 160 \quad \text{g/kg}
\end{aligned}
$$

Physical dimensions of the heat exchanger are presented in figure 4.

The convection coefficients are calculated with the correlations developed for the forced convection over the vertical plate [Incropera, Chapter 7.2].

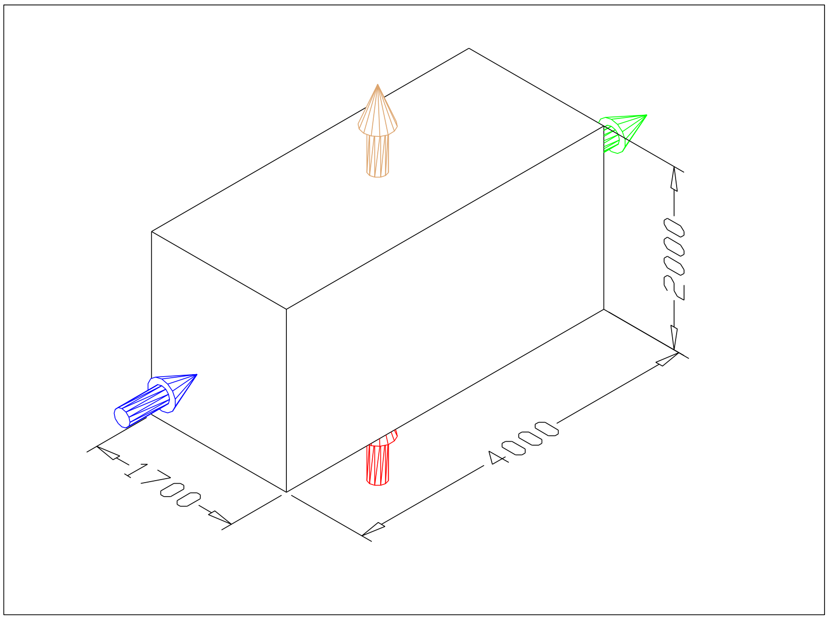

FIGURE 4. Sample heat exchanger

Calculated values for the current exchanger and operating conditions are:

$$
\begin{array}{lcrl}
h_{cold} & = & 56.6 & \mathrm{W/m^2K} \\
h_{hot} & = & 42.9 & \mathrm{W/m^2K} \\
\lambda & = & 20 & \mathrm{W/mK} \\
s & = & 0.5 \cdot 10^{-3} & \mathrm{m} \\
\Rightarrow \mu & = & 24,3 & \mathrm{W/m^2K}
\end{array}
$$

The total heat surface area for the current configuration is 816 m^2.
Using equation (1) and solving it iteratively we get the results:

$$
\begin{array}{lcrl}
T_{cold,out} & = & 55,2 & {}^{o}\mathrm{C} \\
T_{hot,out} & = & 67,8 & {}^{o}\mathrm{C} \\
f & = & 0.953 & \\
\Rightarrow \Phi & = & 636,6 & \mathrm{kW}
\end{array}
$$

3. Differential equations

Here we develop a numerical scheme based directly on the differential equations govering the heat flows. In this case both fluid flows are assumed to remain under the non-condensing conditions and due to that mass transfer is not to be modelled.

Consider first the differential equations of two dimensional heat conduction in two domains below and above the plate separating the air flows

$$
(4) \qquad \begin{aligned} u_t &= \alpha \Delta\, u - \langle v, \nabla u \rangle - \mu\,(u - \tilde{u})\ , \\ \tilde{u}_t &= \tilde{\alpha} \Delta\, \tilde{u} - \langle \tilde{v}, \nabla \tilde{u} \rangle - \tilde{\mu}\,(\tilde{u} - u)\ . \end{aligned}
$$

Here u ($\tilde{u}$) is the temperature of the cold (hot) air, and v, $\tilde{v}$ are the velocities of the flow. In the case of cross flows we take $v = (v_c, 0)$, $\tilde{v} = (0, v_h)$.

When the velocities are large and temperature differences significant, we can omit the horizontal diffusion, i.e., set $\alpha = \tilde{\alpha} = 0$. Then at the equilibrium we have

$$
(5) \qquad \begin{aligned} v_c\, u_x(x,y) + \mu\,(u(x,y) - \tilde{u}(x,y)) &= 0\ , \\ v_h\, \tilde{u}_y(x,y) + \tilde{\mu}\,(\tilde{u}(x,y) - u(x,y)) &= 0\ , \end{aligned}
$$

in the domain $(x,y) \in (0, L_x) \times (0, L_y)$.

Further, the inflow temperatures are constants, so we have the boundary conditions

$$
u(0,y) = T_c\ , \qquad \tilde{u}(x,0) = T_h\,.
$$

The velocities $v, \tilde{v}$ represent the heat capacity rates for the flows divided evenly over the heat exhanger plate surfaces. Factor μ is overall heat transfer coefficient calculated according to the equation (3).

Note

Since $u(0,y) = T_c$ is known, we can solve

$$
\tilde{u}(0,y) = e^{-\frac{\tilde{\mu}}{v_h}\, y}\, T_h + \tfrac{\tilde{\mu}}{v_h} \int_0^y e^{-\frac{\tilde{\mu}}{v_h}\,(y-\xi)}\, u(0,\xi)\, d\xi\ .
$$

Then we find easily the dew point ($\tilde{u}(0, y_{dew}) = T_{dew}$) on the $x = 0$ – boundary:

$$
y_{dew} = \frac{v_h}{\tilde{\mu}}\, \ln\left(\frac{T_h - T_c}{T_{dew} - T_c} \right)\ .
$$

Similarly, since $\tilde{u}(x,0)$ is known, we could solve $u(x,0)$ explicitly.

4. Discretization

Inside the rectangle we consider numerical approximation in a uniform grid consisting of points $(i\,h_x, j\,h_y)$, $i = 0, 1, \ldots, m$, $j = 0, 1, \ldots, n$, where $h_x = L_x/m$, $h_y = L_y/n$.

Let us denote $u_{i,j} \approx u(i\,h_x, j\,h_y)$ and similarly for $\tilde{u}$. Using central differences and averages at $(\,(i + \frac{1}{2})\,h_x\,,\, j\,h_y\,)$ and $(\,i\,h_x\,,\,(j + \frac{1}{2})\,h_y\,)$ we get the equations

$$
(6) \qquad \begin{aligned} v_c\, \frac{u_{i+1,j} - u_{i,j}}{h_x} + \mu \left(\frac{u_{i+1,j} + u_{i,j}}{2} - \frac{\tilde{u}_{i+1,j} + \tilde{u}_{i,j}}{2} \right) &= 0\ , \\ v_h\, \frac{\tilde{u}_{i,j+1} - \tilde{u}_{i,j}}{h_y} + \tilde{\mu} \left(\frac{\tilde{u}_{i,j+1} + \tilde{u}_{i,j}}{2} - \frac{u_{i,j+1} + u_{i,j}}{2} \right) &= 0\ . \end{aligned}
$$

At the inflow boundaries we extrapolate: $\tilde{u}(\frac{1}{2}h_x, jh_y) \approx \frac{3}{2}\,\tilde{u}_{1,j} - \frac{1}{2}\,\tilde{u}_{2,j}$ and $u(ih_x, \frac{1}{2}h_y) \approx \frac{3}{2}\,u_{i,1} - \frac{1}{2}\,u_{i,2}$. Another option would be to use the explicit solutions for $\tilde{u}(0,y)$ and $u(x,0)$ (see the note above) in the averages.

Matrix form

Let us write the unknowns into $m \times n$ matrices: $U = [u_{i,j}]$, $\tilde{U} = [\tilde{u}_{i,j}]$. Then we get

$$
\begin{aligned}
v_c\, D_x\, U + \mu\, M_x\, U - \mu\, E_x\, \tilde{U} &= (\tfrac{v_c}{h_x} - \tfrac{\mu}{2})\, T_c\, J_x\ , \\
v_h\, \tilde{U}\, D_y^T + \tilde{\mu}\, \tilde{U}\, M_y^T - \tilde{\mu}\, U\, E_y^T &= (\tfrac{v_h}{h_y} - \tfrac{\tilde{\mu}}{2})\, T_h\, J_y^T\ ,
\end{aligned} \tag{7}
$$

where

$$
D_x = \begin{bmatrix} 1 & & & \\ -1 & 1 & & \\ & \ddots & \ddots & \\ & & -1 & 1 \end{bmatrix}, \quad M_x = \begin{bmatrix} \frac{1}{2} & & & \\ \frac{1}{2} & \frac{1}{2} & & \\ & \ddots & \ddots & \\ & & \frac{1}{2} & \frac{1}{2} \end{bmatrix}, \quad E_x = \begin{bmatrix} \frac{3}{2} & -\frac{1}{2} & & \\ \frac{1}{2} & \frac{1}{2} & & \\ & \ddots & \ddots & \\ & & \frac{1}{2} & \frac{1}{2} \end{bmatrix},
$$

and J_x has ones on the first row, otherwise it is zero. D_y, M_y, E_y, J_y are obtained similarly (but they may be of different size).

Vector form

Now, write the unknowns in long vectors: $\underline{u} = \begin{bmatrix} u_{1,1} \\ u_{2,1} \\ \vdots \\ u_{m,n} \end{bmatrix}$, $\underline{\tilde{u}} = \begin{bmatrix} \tilde{u}_{1,1} \\ \tilde{u}_{2,1} \\ \vdots \\ \tilde{u}_{m,n} \end{bmatrix}$, i.e., the columns of U (respectively $\tilde{U}$) on top of each other. Then we get the linear system

$$
A \begin{bmatrix} \underline{u} \\ \underline{\tilde{u}} \end{bmatrix} = b\,,
$$

where

$$
A = \begin{bmatrix} I \otimes (v_c\, D_x + \mu\, M_x) & -\mu\, I \otimes E_x \\ -\tilde{\mu}\, E_y \otimes I & (v_h\, D_y + \tilde{\mu}\, M_y) \otimes I \end{bmatrix},
$$

and b is obtained by organizing the right hand sides of (7) accordingly.
In `matlab` we set these up as sparse matrices using `kron` for the Kronecker product $\otimes$. Then we use the direct sparse solver.

5. Numerical results

We apply the equations above with the same parameter values as in earlier solution with the NTU-method.

$$
\begin{aligned}
&T_c = 30^{\circ}\mathrm{C}, \quad L_x = 4, \quad v_c = 121.4, \quad m = 60, \quad h_x = \tfrac{1}{15}, \\
&T_h = 85^{\circ}\mathrm{C}, \quad L_y = 2, \quad v_h = 90.6, \quad n = 40, \quad h_y = \tfrac{1}{20}.
\end{aligned}
$$

The overall heat transfer coefficient is $\mu = \tilde{\mu}$=24.3 W/m^{2}K.

Figure 5 shows the temperature profiles in the equilibrium solved with the above listed parameters.

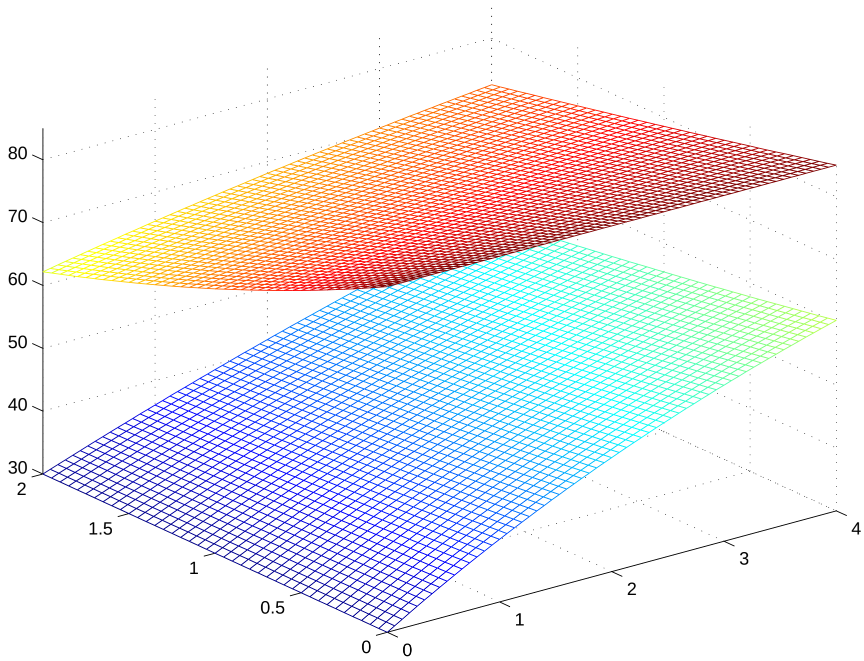

FIGURE 5. Temperature for the hot and cold air stream across the heat exchanger slot

At the outflow boundaries we get the following average temperatures, which can be compared with the temperatures got from NTU-solution with the assumption of counter-flow conditions:

$$\begin{aligned} T_{cold,out} &= 55,5^{o}\text{C} \\ T_{hot,out} &= 67,9^{o}\text{C} \end{aligned}$$

The power of the heat exchanger is $\Phi = 632.3\,\text{kW}$.

6. CONCLUSIONS

The model for cross-flow heat exchanger was developed and the results were compared with the results calculated with the traditional dimensioning method (NTU). The results were as follows:

Parameter	NTU-method	Developed method
$T_{cold,out}$	55.2 °C	55.5 °C
$T_{hot,out}$	67.8 °C	67.9 °C
Φ	636.6 kW	632.2 kW

As seen above the model seems to work well under the specified operation conditions.

The future actions to develop the model further is to focuse on the following areas:

(1) Additional test runs for different heat exchanger geometries
(2) Modelling of the material and fluid properties
(3) Heat and mass transfer model for the condensing conditions
(4) Model for the multi-pass heat exchanger configuration

List of Symbols

Symbols

f	correction factor to fix the error got from the assumption of counter-flow temperatures	[-]
h	convection heat transfer coefficient	[W/m^{2}K]
q	flow	[unit/s]
s	thickness	[m]
v	velocity, heat capacity rate	[J/Ks]
x	absolute water content of the air	[kg/kg]
A_{tot}	total effective heat surface area	[m^2]
T	Temperature	[K]
ΔT_{ln}	log mean temperature difference	[K]
λ	conductivity	[W/mK]
μ	overall heat transfer coefficient	[W/m^{2}K]
Φ	power	[W]
$1, 2$	indices	[—]

Subscripts

c, cold	cold fluid flow	[-]
m	mass	[kg]
$\tilde{}$, h, hot	hot fluid flow	[-]

References

Incropera, Frank P. & DeWitt, David P., 1990. Fundamentals of Heat and Mass Transfer. 3. p. Indiana. John Wiley & Sons.

VDI-Heat Atlas, 1993. 6.p. Düsseldorf. VDI-Verlag GmbH.