Description of the course

A linear system is described by the (linear) differential equation

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t), \qquad t \geq 0,\ x(0) = x_0 \\ y(t) &= Cx(t) + Du(t), \end{aligned}$$

where u is the input (control), x is the state, and y is the output (observations). These functions take their values in the linear space U, X, and Y, respectively. Furthermore, A, B, C, and D are linear mappings between the appropriate spaces.

The central theme within systems theory is to design u such that y and/or x has a desired behavior. A typical example is stabilization: Design an input u such that $x(t) \to 0$ for $t \to \infty$.

Motivated by the engineering applications, the construction of the control function u should be robust under small errors in the initial condition x_0 and the system parameters, A, B, C, and D. A control designed by feedback achieves this goal. Feedback means that we do not calculate u only based on x_0, A, B, C, and D, but also on the current state. This results in a control of the form $u(t) = Fx(t)$, and the question of stabilization turns into a question of finding F.

If the spaces U, X, and Y are finite-dimensional, then the system is called finite-dimensional. Otherwise, we have an infinite-dimensional linear system. Since we assume no prior knowledge of systems theory, we begin with some of the highlights of finite-dimensional systems theory. Linear algebra and functional analysis play an important role in finite- and infinite-dimensional systems theory, respectively. The necessary background in functional analysis will be listed in the appendix and the references of the lecture notes.

Infinite-dimensional linear systems appear naturally when studying control problems for systems modelled by linear partial differential equations. Further, many physical models have a conserved quantity, like mass or energy. It is very beneficial to use this quantity in the design of a controller. This has led to the class of port-Hamiltonian systems, which is one of the classes we treat in more detail. Port-Hamiltonian systems with boundary control and observations can be treated with only a minimal of mathematical technicalities. Topics that will be studied in this course are:

- Controllability and stabilizability of finite-dimensional systems
- Strongly continuous semigroups
- Stabilizability for infinite-dimensional systems
- Boundary control for partial differential equations
- Transfer functions
- Stabilizability of port-Hamiltonian systems
- Well-posedness for linear infinite-dimensional systems