Matemaattisen systeemiteorian kurssi

TTY:llä järjestetään johdantokurssi ääretönulotteiseen systeemiteoriaan osana kansainvälistä verkkoseminaaria. Kurssin kesto on lokakuun puolivälistä helmikuuhun ja se koostuu viikottaisista videoluennoista ja näihin liittyvistä harjoitustehtävistä.

Kurssia suositellaan etenkin kaikille differentiaaliyhtälöistä, funktionaalianalyysistä tai matemaattisesta mallinnuksesta kiinnostuneille. Aihe liittyy myös kurssiin **MAT-51330 Jakautuneet järjestelmät** (ei vaadita esitiedoiksi). Tarkempi kuvaus aihepiiristä löytyy kurssin kotisivuilta `http://math.tut.fi/sysgroup/ISEM`.

Ilmoittautumiset kurssille 15.10. mennessä suoraan kurssin assistentille Lassi Paunoselle (lassi.paunonen@tut.fi, Td319). Myös kaikki tiedustelut kurssista voi suunnata hänelle.

Suositeltavina esitietovaatimuksina on perustiedot funktionaalianalyysistä, MAT-41140 Johdatus funktionaalianalyysiin tai vastaava. Toisaalta kurssista voi suorittaa pelkän alkuosan, joka käsittelee äärellisulotteisten järjestelmien teoriaa. Tällöin esitiedoiksi riittävät matriisilaskennan perusteet. Kurssi kelpaa jatko-opinnoiksi.

A Course on Mathematical Systems Theory

This semester TUT organizes an introductory webcourse on infinite-dimensional systems theory as a part of an international internet seminar. The course lasts from the middle of October to February and it consists of weekly online lectures and associated exercises.

The course is recommended especially to everyone interested in differential equations, functional analysis or mathematical modeling. The topic is also related to the course **MAT-51336 Distributed Parameter Systems** (not required as a prerequisite). A detailed description can be found on the website at `http://math.tut.fi/sysgroup/ISEM`.

Enrolments for the course before October 15th to Lassi Paunonen (lassi.paunonen@tut.fi, office Td319). All other inquiries about the course can also be directed to him.

Recommended prerequisite for the course is a basic knowledge of functional analysis. On the other hand, it is also possible to complete only the first part of the course consisting of finite-dimensional systems theory. In this case the basics of linear algebra are sufficient. The course is suitable for postgraduate studies.

Description of the course

A linear system is described by the (linear) differential equation

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t), \qquad t \geq 0, \ x(0) = x_0 \\ y(t) &= Cx(t) + Du(t), \end{aligned}$$

where u is the input (control), x is the state, and y is the output (observations). These functions take their values in the linear space U, X, and Y, respectively. Furthermore, A, B, C, and D are linear mappings between the appropriate spaces.

The central theme within systems theory is to design u such that y and/or x has a desired behavior. A typical example is stabilization: Design an input u such that $x(t) \to 0$ for $t \to \infty$.

Motivated by the engineering applications, the construction of the control function u should be robust under small errors in the initial condition x_0 and the system parameters, A, B, C, and D. A control designed by feedback achieves this goal. Feedback means that we do not calculate u only based on x_0, A, B, C, and D, but also on the current state. This results in a control of the form $u(t) = Fx(t)$, and the question of stabilization turns into a question of finding F.

If the spaces U, X, and Y are finite-dimensional, then the system is called finite-dimensional. Otherwise, we have an infinite-dimensional linear system. Since we assume no prior knowledge of systems theory, we begin with some of the highlights of finite-dimensional systems theory. Linear algebra and functional analysis play an important role in finite- and infinite-dimensional systems theory, respectively. The necessary background in functional analysis will be listed in the appendix and the references of the lecture notes.

Infinite-dimensional linear systems appear naturally when studying control problems for systems modelled by linear partial differential equations. Further, many physical models have a conserved quantity, like mass or energy. It is very beneficial to use this quantity in the design of a controller. This has led to the class of port-Hamiltonian systems, which is one of the classes we treat in more detail. Port-Hamiltonian systems with boundary control and observations can be treated with only a minimal of mathematical technicalities. Topics that will be studied in this course are:

- Controllability and stabilizability of finite-dimensional systems
- Strongly continuous semigroups
- Stabilizability for infinite-dimensional systems
- Boundary control for partial differential equations
- Transfer functions
- Stabilizability of port-Hamiltonian systems
- Well-posedness for linear infinite-dimensional systems