Local Positioning with Parallelepiped Moving Grid

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Personal positioning

for example, a mobile handset

sources of positioning information:
- navigation equipment: GPS, IMU
- cellular network, WLAN, Bluetooth, . . .
- digital compass, step counter, barometer
- aiding data (altitude, cell sector, . . .)
- digital maps
- etc.
Challenges

- large linearization errors
- scarce measurements – underdetermined
- non-normal noise structure
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→ exploit the time dependency
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- large linearization errors
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→ make use of **all** available information
→ exploit the time dependency
→ allow flexible measurement models
Discrete nonlinear filtering problem

Distribution of initial state $x_0$ given.

Motion model:

$$x_{k+1} = f_k(x_k) + w_k$$

Measurement model:

$$y_k = h_k(x_k) + v_k$$

Optimal solution: recursive Bayesian filter
Recursive Bayesian filter

Prediction step:

\[ p(x_k \mid Y_{k-1}) = \int_{\mathbb{R}^d} p(x_k \mid x_{k-1}) p(x_{k-1} \mid Y_{k-1}) \, dx_{k-1} \]

- motion model
- previous
Recursive Bayesian filter

Prediction step:
\[ p(x_k | Y_{k-1}) = \int_{\mathbb{R}^d} p(x_k | x_{k-1}) p(x_{k-1} | Y_{k-1}) \, dx_{k-1} \]

Update step:
\[ p(x_k | Y_k) \propto p(y_k | x_k) p(x_k | Y_{k-1}) \]

Recursive Bayesian filter

Prediction step:

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In general, exact solution requires infinite time and memory!
Numerical pdf approximations

- True pdf
- Point mass
- Grid mass
- Gaussian
- Monte Carlo
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Moving grid algorithm

1. prior approximation
Moving grid algorithm

1. prior approximation
2. measurement likelihood
Moving grid algorithm

1. prior approximation
2. measurement likelihood
3. approximate likelihood
Moving grid algorithm

1. prior approximation
2. measurement likelihood
3. approximate likelihood
4. multiply to get posterior
Moving grid algorithm

1. prior approximation
2. measurement likelihood
3. approximate likelihood
4. multiply to get posterior
5. propagate with motion model
Moving grid algorithm

1. prior approximation
2. measurement likelihood
3. approximate likelihood
4. multiply to get posterior
5. propagate with motion model
6. repeat from step 2 . . .
Example run
Example run
Example run
Grid accuracy

The 2D error of the moving grid filter mean estimate compared to the cell radius
How to compare filters?

Ideally: compare the posterior distribution to the ideal one

In practise we compute the mean and covariance estimates

- error from true track
- error from reference track
- consistency (error vs. estimated covariance)
## Some numbers

<table>
<thead>
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<th></th>
<th>mean error</th>
<th>95 % CERP</th>
<th>inconsistent</th>
<th>time</th>
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<tbody>
<tr>
<td>grid</td>
<td>200 m</td>
<td>574 m</td>
<td>&lt;0.1 %</td>
<td>100</td>
</tr>
<tr>
<td>EKF</td>
<td>105 m</td>
<td>370 m</td>
<td>4 %</td>
<td>1</td>
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<td>SMC, 10k</td>
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<td>260 m</td>
<td>1 %</td>
<td>30</td>
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<tr>
<td>SMC, 2M</td>
<td>75 m</td>
<td>255 m</td>
<td>&lt;0.1 %</td>
<td>10000</td>
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</tbody>
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Conclusions

- grid filter gives rough but reliable results
- fair and expressive comparison of nonlinear filters still an open problem
  - how to weigh the overall score?
  - compare to true track or reference track?
  - accuracy vs. reliability
  - accuracy vs. computation time?
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Questions?