Using Unlocated Fingerprints in Generation of WLAN Maps for Indoor Positioning

Matti Raitoharju, Toni Fadjukoff, Simo Ali-Löytty, Robert Piché
Tampere University of Technology
Tampere, Finland
matti.raitoharju@tut.fi

Abstract—This paper presents five methods for generation of WLAN maps for indoor positioning using crowdsourced fingerprints. A fingerprint is assumed to contain identifiers of WLAN access points, received signal strength values and, if the fingerprint is collected outdoors, a GPS position. The proposed methods use the fingerprints’ information to generate a WLAN map that contains estimated access point locations. Two of the proposed methods use RSS values in access point location estimation. In our evaluation with simulations and with real data, the Access Point Least Squares method, which does not use RSS information, is the fastest and its accuracy is as good as more complex methods that use RSS information.

I. INTRODUCTION

WLAN (Wireless Local Area Network) is the most commonly used method for enabling wireless network connections in mobile devices. The coverage area of a single WLAN AP (access point) is on the order of tens of meters. Although WLAN was not designed for positioning purposes, the abundance of APs and the prevalence of WLAN receivers in mobile devices makes positioning using WLAN an alternative to GPS (Global Positioning System), especially for positioning indoors (airports, malls etc.) where GPS is often unusable.

Most of proposed WLAN positioning systems belong to either of following two categories

1) Fingerprinting methods: In fingerprinting the area of interest is mapped by identifying which WLAN stations can be received and their signal strengths in known positions; each such measurement is called a fingerprint (FP). A mobile device’s position can then be computed by comparing its received WLAN signals with those in database. A survey of different fingerprinting methods is done in [1].

2) Network topology modeling methods: In these methods the measured FPs are processed to estimate parameters of the network, for example coverage areas, AP positions and signal attenuation models. An example of a coverage area method is presented in [2].

The large scale collection of fingerprints may be done in massive and expensive data collection campaigns. An alternative is for measurements to be collected by normal users with their own equipment. This method is called crowdsourcing. Crowdsourcing should require as little as possible user interaction and the data sent in by users may have a lot of errors. The WLAN scans may be done automatically and GPS may be used for receiving absolute position outdoors, but indoors there usually is not position information available.

In this paper we study methods to generate WLAN maps that contain estimated positions for WLAN APs for position using fingerprints collected with GPS enabled devices. A FP is assumed to contain unique identifiers for WLAN APs that are received, possibly received signal strength (RSS) values and GPS positions when the measurements are done outdoors. Figure 1 shows an example of a situation where FPs contain a list of received APs located inside the square building, and only the FPs outside the building have location information. The goal of our methods is to estimate the AP locations. The methods are developed considering that they should be applicable to a building meaning that they can handle some hundreds of APs simultaneously. This does not restrict the applicability of the presented methods on global scale as GPS is available between buildings and the mapping may be then divided into building scale subproblems.

![Figure 1. APs, located FPs and unlocated FPs in and around a square building](image-url)
The problem is similar to localization of sensors in sensor networks. These methods are covered for example in [3]. The main difference to sensor localization methods is that in addition to AP locations we have significant numbers of FPs that do not have location information and are not interesting for us. Most of evaluated methods are designed to be such that the distribution of FPs should not affect results e.g. in buildings where there are more and less used areas.

The rest of the article is organized as follows. In Section II the different sources of measurements (input data of algorithms) are presented. In Section III the different algorithms for doing AP positioning are presented. A simple positioning method that uses AP locations is presented in Section IV. In Section V the performance of algorithms in AP and user positioning is evaluated and Section VI concludes the article.

II. MEASUREMENTS

In the following sub sections the used measurement sources are presented. The measurements are related to WLAN and GPS systems, which both are available in modern smartphones.

A. Absolute Position

Absolute position is assumed to be received using GPS. GPS receivers provide position estimates with error of a couple of meters in good signal conditions. In this paper we assume that when the GPS position is available it is exact. In real situations the GPS may have hundreds of meters of error, especially close to buildings where only a few GPS signals are available. For real use of the proposed methods we assume that the quality of GPS is monitored and that bad GPS positions have been discarded.

B. Connectivity

Connectivity is determined by checking which APs are received at FPs. Any APs that are received simultaneously have overlapping coverage areas.

C. Received Signal Strength

RSS (received signal strength) measurements are commonly used to do localization. RSS measurements can be converted to range measurements using a simple path loss model [4]

$$\frac{\text{RSS}}{\text{RSS}_0} = 10^{\alpha \log_{10} ||r - x|| + \varepsilon},$$

where RSS$_0$ is the signal strength at range of 1 m from the AP, $\alpha$ is the attenuation factor, $r$ is the receiver position, $x$ the AP location and $\varepsilon$ is the measurement error. In vacuum the value of $\alpha$ is 2, but in realistic situations it should be more. The distance to AP can be explicitly solved from (1) and is

$$||r - x|| = 10^{\frac{\text{RSS} - \text{RSS}_0}{10 \alpha}},$$

III. AP POSITIONING ALGORITHMS

The goal of AP positioning algorithms is to produce a map of APs using FPs some of which do not have position information. In our case we assume that the measurement come in as a batch and may be treated simultaneously.

A. Mean

In the mean algorithm the position estimate of AP is computed by taking the mean of the FP positions where AP was received. If a FP does not have GPS information the position is the mean of the estimates of the received APs. Because the update is done one FP at a time the outcome of algorithm is dependent on the order of FPs. The update of a position of a AP location may be expressed as follows

$$x_{i,t} = \frac{(t-1)x_{i,t-1} + r_t}{t},$$

where $x_{i,t}$ is the position estimate of an AP when $t$ measurements have been processed. The position is

$$r_t = \begin{cases} r_{GPS} & \text{if GPS available} \\ \sum_{j \in \text{FP}} x_{j,t-1} / m & \text{if GPS not available} \end{cases},$$

where $m$ is the number of APs that were in FP and already had an estimated position. The position is not computed and $t$ is not updated if there is no GPS measurement or position estimate for any AP in the FP. The Mean algorithm is simple to implement and has fast runtime and as such it may be considered to be a baseline method that other algorithms should outperform in accuracy.

B. Gauss-Newton

The Gauss-Newton (GN) method is named after the Gauss-Newton optimization algorithm [5]. In GN method we use all the data to solve the AP positions and in addition to AP positions the actual FP positions are solved as a side product. The model uses distances between FP and AP locations as given by (2). The optimization goal is to find such locations to FPs and APs that their distances are as close as possible to distances computed by RSS values. This is done by minimizing the sum

$$\sum_{i,j} \left( \frac{\text{RSS}_{i,j} - \text{RSS}_0}{\text{RSS}_0} - ||x_i - r_j|| \right)^2,$$

where RSS$_{i,j}$ is the received signal strength of $i$th AP in $j$th FP. The minimization is done using the Gauss-Newton algorithm.

In Gauss-Newton optimization the sum of squares is minimized iteratively by taking following steps

$$\hat{z}_i = \hat{z}_{i-1} + \beta J_{\hat{z}_{i-1}}^{-1} (y - h(\hat{z}_{i-1})), \quad \text{or} \quad J_{\hat{z}_{i-1}}^{-1} = \frac{\partial h(\hat{z}_{i-1})}{\partial \hat{z}_{i-1}} \text{and} \beta \text{ is a positive scalar. The \textbackslash operator is used as it is in Matlab i.e. if } J_{\hat{z}_{i-1}} \text{ is fully determined it solves the}$$
system of linear equations, if it is overdetermined the result is the least squares solution and if it is underdetermined it returns one of the solutions. The vector $\hat{z}$ contains all AP and FP location estimates concatenated and the vector $y$ contains all distances computed with (2) and the measurement function has all the corresponding distances as functions of FP and AP locations
\[ \|x_i - r_j\|. \] (7)

The GN requires an initial value $\hat{z}_0$ for AP and FP positions and may converge to different local minima depending on the initial value.

The part of the Jacobian corresponding to FP and AP parts are
\[ J^{FP}_i = \frac{(r_j - x_i)^T}{\|r_j - x_i\|}, \] (8)
\[ J^{AP}_i = -\frac{(r_j - x_i)^T}{\|r_j - x_i\|}. \] (9)

When the FP has a GPS position, the $r_j$ is fixed and the Jacobian does not contain elements corresponding to this variable and if $\|r_j - x_i\| = 0$ the corresponding Jacobian element is assigned the value 0.

The parameter $\beta$ is chosen at each iteration to ensure that the sum of squares of $y - h(\hat{z}_{i-1})$ decreases. In our implementation it is set to one at the beginning of each iteration and halved until a value is found such that the objective function decreases. We stop the iteration when all corrections $\|x_{i,k} - x_{i,t-1}\|$ are smaller than a predetermined threshold.

C. Gauss-Newton Max Range

The Gauss-Newton max range (GNMax) method is based on the same iterative optimization method as the GN method, but the model is different. Whereas in Gauss-Newton the positions of unlocated FPs are estimated and the number of equations grows, as the number of FPs grows. In GNMax the number of equations is limited to be the number of AP connections. If two APs are received at once, then the distance between APs is less than or equal to the sum of distances from the FP location to both APs. This may be expressed with the triangle inequality
\[ \|x_i - x_k\| = \|(x_i - r_j) + (r_j - x_k)\| \leq \|x_i - r_j\| + \|r_j - x_k\|. \] (10)

While the GN tries to find such APs and FPs that the distances between those match as well as possible to measurement values, the GNMax finds such locations of APs that they are not too far away from each other.

If the FP and the APs are collinear then (10) is an equality. If there are more than one FPs where we receive same APs then the measurement used is the one where the sum of distances computed with RSS values is smallest. In the optimization phase we discard distances where estimates of APs are closer to each other than the measurement indicates. This way the estimation uses only the RSS measurements only to limit the maximum distance between APs. The optimization of measurements is done as in GN, but if two APs are current estimate are closer than the sum of distances, the measurement is neglected. The Jacobian between two APs are
\[ J^{AP}_k = \frac{(x_k - x_i)^T}{\|x_k - x_i\|}, \] (11)
\[ J^{AP}_i = -\frac{(x_k - x_i)^T}{\|x_k - x_i\|}, \] (12)

if
\[ \|x_k - x_i\| > \frac{\text{RSS}_{i,j} - \text{RSS}_{k,j}}{\text{RSS}_{k,j}} + 10^\frac{\text{RSS}_{k,j} - \text{RSS}_{i,j}}{\text{RSS}_{i,j}} \] (13)

otherwise the distance is not used in update (6).

The GPS measurements are taken into account by using up to three FPs with location information. If there are more than three FPs for an AP with GPS positions we choose three points from the convex hull of the points that produce the largest triangle. This should produce a good geometry for measurements.

D. Access Point Least Squares

Access Point Least Squares (APLS) method may be thought as a spring model, where two APs that are received simultaneously or a AP and a GPS location have a spring pulling them together. The model behind the method is explained more mathematically next.

If a device at $r$ receives signals from two WLAN APs it is located inside of the intersection of the coverage areas. This may be written as
\[ r = \hat{x}_{i,j} + \varepsilon_{i,j}, \] (14)
where $\hat{x}_{i,j} = \hat{x}_{j,i}$ is the ”center” of the intersection area and $\varepsilon_{i,j} = \varepsilon_{j,i}$ is the position displacement from the center. The center of the intersection may be written using translation vectors $v_{i,j}$ and $v_{j,i}$ that represent the relative positions of the intersections compared to APs positions. The variables used in APLS are presented in Figure 2. By doing the following subtraction, the FP location is eliminated
\[ r = \hat{x}_{i,j} + \varepsilon_{i,j} = x_i + v_{i,j} + \varepsilon_{i,j} = x_j + v_{j,i} + \varepsilon_{j,i} \Rightarrow 0 = x_i - x_j + v_{i,j} - v_{j,i}. \] (15)

If the translation vectors are independent random variables then the variance of difference is
\[ \text{var}(v_{i,j} - v_{j,i}) = \text{var}(v_{i,j}) + \text{var}(v_{j,i}). \] (17)

If we consider only pairwise relationships between the positions of the APs, the AP locations may be estimated by finding a least squares solution for the following equations:
\[ x_i - x_j = 0, \text{ for all } i \text{ and } j \text{ connected} \]
\[ x_i = r_i, \text{ for all } i \text{ with GPS positions}, \] (18)
where $\sigma_i$ is the mean of GPS positions of FPs that has $i$th AP. These equations may be written in matrix form

$$
\begin{bmatrix}
D \\
G
\end{bmatrix}
\begin{bmatrix}
x_1^T \\
\vdots \\
x_n^T
\end{bmatrix}
= 
\begin{bmatrix}
0 \\
0 \\
\vdots \\
0 \\
r_1^T \\
\vdots \\
r_k^T
\end{bmatrix}, 
$$

(19)

where $D$ contains all the differences and $G$ all the GPS measurements. To find the least squares solution to this set of equations assume that the equations are linearly independent and let $W_D$ denote the diagonal matrix containing the reciprocals of variance of AP location differences $1/\sigma_j^2$ on the diagonal and similarly $W_G$ containing the reciprocals of GPS variances. The weighted least squares solution is now

$$
\begin{bmatrix}
x_1^T \\
\vdots \\
x_n^T
\end{bmatrix}
= 
\begin{bmatrix}
D^T \\
G^T
\end{bmatrix}
\begin{bmatrix}
W_D & 0 \\
0 & W_G
\end{bmatrix}
\begin{bmatrix}
D \\
G
\end{bmatrix}^{-1}
\begin{bmatrix}
r_1^T \\
\vdots \\
r_k^T
\end{bmatrix}, 
$$

(20)

and

$$
G^T W_G G = B = \text{diag}(\phi_1, \ldots, \phi_n), \text{ where } \phi_i = \begin{cases} 
\frac{1}{\sigma_{i,\text{GPS}}} & \text{if any FP has GPS position } \\
0 & \text{otherwise} 
\end{cases} 
$$

(23)

and

$$
\begin{bmatrix}
\phi_1 r_1^T \\
\vdots \\
\phi_n r_k^T
\end{bmatrix} = \bar{y} = 
\begin{bmatrix}
\phi_1 r_1^T \\
\vdots \\
\phi_n r_k^T
\end{bmatrix}, 
$$

(25)

The solution is now simply

$$
\begin{bmatrix}
x_1^T \\
\vdots \\
x_n^T
\end{bmatrix}
= (A + B)^{-1} \bar{y}. 
$$

(26)

In APLS all the variances between APs are set to be equal. We found that the AP positioning gives best results when the GPS weight $1/\sigma_{i,\text{GPS}}^2$ is set to be $\theta A_{i,j}$, where $\theta$ is a scalar parameter. In Figure 3 the different weighting schemes are evaluated. Dots represent the AP positions, stars are GPS positions and circles are estimated AP positions. In leftmost the weight between GPS measurement and APs is low. In center the GPS weight is infinite and on right an optimal weight is used. The optimal value of $\theta$ is discussed in Section V.
E. Variance Access Point Least Squares

Variance APLS (VARAPLS) takes the RSS values between APs into account when computing the variances, as follows

\[ \sigma_{i,j}^2 = (\min ||r - x_j|| + ||r - x_i||)^2. \] (27)

IV. USER POSITIONING

As the ultimate goal of the research is to position user and not the AP we need to evaluate AP positioning algorithms in user positioning. For user positioning we compute the mean of the five strongest APs received

\[ r = \frac{\sum_{i\in \text{five strongest}} x_i}{5}. \] (28)

This method is used as it uses only AP locations and in results section we show that if the AP positions are correctly located it provides rather good positioning performance.

V. RESULTS

First we look at finding the optimal \( \theta \) for APLS and VARAPLS. We varied the parameter value and did positioning tests similar to one in Figure 3. In Figure 4 the effect of \( \theta \) on AP localization is presented. Optimal values were found to be 1.63 for APLS and 1.58 for VARAPLS.

In Figure 5 is shown the effect of different parameters to runtime and accuracy of the methods in a simulated environment. In simulations AP:s were randomized inside a box of area 100 m \( \times \) 100 m. The FPs were randomized on a slightly larger box that exceeded the AP box by 10% in all directions. The FPs that were located outside the AP box were considered to have exact position information. We investigated the influence of five parameters on the performance of the presented methods. The tests were done using combinations of different parameters. The GN methods require an initial estimate of AP positions. As initial estimate we used the mean position of all GPS FPs for all of APs and FPs. The methods marked with prior in Figure 5 use APLS estimate as initial estimate for APs. As the stopping condition for GN methods we used that all APs move less than 10\(^{-4}\) m in (6).

In first row of Figure 5 parameter \( \sigma \) is the standard deviation of RSS measurements. The errors were rounded samples from zero mean normal distribution. In GN methods the RSS value is transformed into distance. The error in distance does not have zero mean. This bias is left uncompensated because to simulate the modeling errors in path loss parameters. The computation time of Gauss-Newton method increases as measurement error increases, because the solution does not have a clear optimum when error increases. In the error dimension we see that GNMax gives worse results than APLS when standard deviation is more than 5 dBm and for GN with prior this limit is 9 dBm. This implies that if the signals are noisy or the RSS model is not accurate the APLS should be used. On other hand, if accurate distance measurements are available these methods have small errors. In [6] the standard deviation of WLAN measurements is studied and results show that the value varies between 1 and 5 when the device does not move, but the received mean may change 4 dBm depending on the orientation of user and in [7] the standard deviation of WLAN measurements was found to be usually more than 10 dBm.

In the second row of Figure 5 the effect of the number of APs is evaluated. In the third row the number of FPs is evaluated. There one should note that the runtime of GNMax gets smaller as the number of FPs increase. In fourth row the range of an AP is evaluated. There we see that the APLS based methods work better the smaller the range is, that is, they work best if the building is big. On the other hand the GN benefits from long ranges of measurements.

In the last row of Figure 5 the effect of nonuniform distribution of FPs is analyz. The abscissa is probability that the FP, instead of being randomly located on whole area, is located inside 20 m wide corridor in the center of the building. All proposed methods suffer from the nonuniform distribution of FPs.

In general we see that GN benefits a lot from using APLS as the prior, whereas GNMax is not so sensitive to the initial condition. The accuracy of the Mean method is worse than the accuracy of any other method. Also the time of execution of GN methods is much greater than APLS and Mean method and our APLS implementation is even faster than the implementation of Mean algorithm.

In Figure 6 routes solved from AP locations estimated by different methods are presented. The building was walked once around outside and then some measurements were done inside without GPS available. The grey dots in the figure represent FPs with GPS positions. All the GPS measurements were made outdoors and the points that are inside of building show
Figure 5. Performance of different methods in simulated environment
Figure 6. Real data positioning done using radiomaps generated with different methods and error distributions.
the effect of errors in GPS measurements. The data had 440 APs and 400 FPs in total of which 120 had GPS position and 280 were taken indoors without GPS. In GN methods we used path loss parameters $\text{RSS}_0 = -40 \, \text{dBm}$ and $\alpha = 3.5$. The boxplots in the lower right corner show 5%, 25%, 75%, 95% quantiles and the mean of error. For the reference track, the AP locations were defined by making measurements in different points inside the building and choosing the AP location to be the mean of the five locations where the AP was received with strongest RSS. APs for the reference map were located using measurements in the same floor where the test track was walked. The times that different methods used were less than 0.05 s for Mean and both APLS variants, for GNMax it took 260 seconds to find an estimate and for GN the time was around 10000 seconds. For GN and GNMax the iterations were stopped when all APs moved less than 2 meters on one iteration. From these results we see that the Mean method does not work at all. The GNMax gives estimates close to the center of the building. APLS, VARAPLS and GN give quite similar performance, where the right side of building was estimated better and the left side worse. APLS had the best mean error of 33 meters, while the reference mean was 11 meters.

VI. CONCLUSIONS

In this paper we presented five methods for localization APs in cases where position information is not always available. Our results show that proposed methods APLS and VARAPLS perform quite similarly and perform well in realistic simulation cases where noise is present. The positioning test with real measurements showed that APLS may be used to generate a map with 440 APs in less than 0.05 s that may be used in positioning at least on a rough scale or as a prior for more complex methods.

ACKNOWLEDGEMENT

This work was supported by Nokia, Inc.

REFERENCES


