Mobile Tracking and Parameter Learning in Unknown Non-line-of-sight Conditions

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- Problem statement
- Bayesian framework
- Rao-Blackwellized particle filtering with parameter learning (RBPF-PL)
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Introduction

- Non Line-of-Sight (NLOS) condition
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- Non Line-of-Sight (NLOS) condition

- State of the art mitigation methods
  - two-step Kalman filter;
  - interactive multiple model + Kalman smoother;
  - grid based Bayesian estimation;
  - modified EKF banks + data fusion;
  - particle filtering;
  - RBPF;
  - particle filtering + UKF
  - Posterior Cramér-Rao lower bound
Introduction

- Non Line-of-Sight (NLOS) condition

- State of the art mitigation methods
  - two-step Kalman filter;
  - interactive multiple model + Kalman smoother;
  - grid based Bayesian estimation;
  - *modified EKF banks + data fusion*;
  - particle filtering;
  - *RBPF*;
  - particle filtering +UKF
  - *Posterior Cramér-Rao lower bound*

**ASSUMPTION:** a complete knowledge of statistics of NLOS errors.

**NOT PLAUSIBLE** in many practical situations.
Problem statement

- **Motivation**
  mobile tracking in *unknown* NLOS conditions
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  mobile tracking in *unknown* NLOS conditions

- **System model**

  \[
  \begin{align*}
  &x_k = \Phi_{k-1}x_{k-1} + w_{k-1} \\
  &s_{i,k} \sim MC(\pi_i, A_i), \\
  &z_{i,k} = h_i(x_k) + v(s_{i,k})
  \end{align*}
  \]

<table>
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<th>Boolean</th>
<th>Mean</th>
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<th>Assumption</th>
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<td>(s_{i,k} = 0)</td>
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<td>(\sigma_n^2)</td>
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Problem statement

- **Motivation**
  mobile tracking in *unknown* NLOS conditions

- **System model**

  \[
  \begin{cases}
  x_k = \Phi_{k-1}x_{k-1} + w_{k-1} \\
  s_{i,k} \sim MC(\pi_i, A_i) \\
  z_{i,k} = h_i(x_k) + v(s_{i,k})
  \end{cases}
  \]

  Boolean  mean  variance  Assumption
  \[s_{i,k} = 0 \quad 0 \quad \sigma_n^2 \quad \sigma_n \text{ known}\]
  \[s_{i,k} = 1 \quad \mu_{\text{NLOS}} \quad \sigma_{\text{NLOS}}^2 + \sigma_n^2 \quad \{\mu_{\text{NLOS}}, \sigma_{\text{NLOS}}\} \text{ fixed but unknown}\]

- **Problem**
  - infer \( p(x_k, s_k, \theta|z_{1:k}) \), where \( \theta = \{\mu_{\text{NLOS}}, \eta\} \) and \( \eta \triangleq \sigma_n^2 + \sigma_{\text{NLOS}}^2 \).
Bayesian framework

Denote $y_k = \{x_k, s_k\}$, the sequential estimation within the Bayesian framework:

$$p(y_{1:k}, \theta|z_{1:k}) \propto p(z_k|y_k, \theta) \cdot p(y_{k-1}|y_k, \theta) \cdot p(y_k|y_{k-1}, z_{1:k-1}) p(y_{1:k-1}|z_{1:k-1})$$

(1)

Suppose at time $t_{k-1}$, we have samples:

$$p(y_{1:k-1}|z_{1:k-1}) \approx \sum_{j=1}^{N} w_{k-1}^j \delta(y_{1:k-1} - y_{1:k-1}^j)$$

(2)

At time $t_k$, with the reception of $z_k$, if the importance density is chosen to factorize as

$$q(y_{1:k}, \theta|z_{1:k}) = q(y_k, \theta|y_{1:k-1}, z_{1:k}) q(y_{1:k-1}|z_{1:k-1})$$

(3)

and the new particles are sampled according to

$$\{y_k^j, \theta^j\} \sim q(y_k, \theta|y_{1:k-1}^j, z_{1:k})$$

(4)

then the weight can be updated as

$$w_k^j \propto \frac{p(y_k^j, \theta^j|z_{1:k})}{q(y_k^j, \theta^j|z_{1:k})} = \frac{p(z_k|y_k^j, \theta^j) p(y_k^j|y_{k-1}^j) p(\theta^j|y_{1:k-1}^j, z_{1:k-1})}{q(y_k^j, \theta^j|y_{1:k-1}^j, z_{1:k})} w_{k-1}^j$$

(5)
RBPF-PL (1/2)

- Factorize the posterior $p(x_k, s_k, \theta|z_{1:k})$ according to Bayes’ rule:

$$p(x_k, s_k, \theta|z_{1:k}) = p(x_k|s_k, \theta, z_{1:k})p(s_k, \theta|z_{1:k}).$$

- If $p(s_k, \theta|z_{1:k})$ is represented by a set of weighted samples $\{s_{j}^k, \theta^j, w_j^k\}_{j=1}^{N}$, then

$$p(x_k|z_{1:k}) \approx \sum_{j=1}^{N} w_j^k p(x_k|s_j^k, \theta^j, z_{1:k}), \quad \text{where} \quad p(x_k|s_j^k, \theta^j, z_{1:k}) \approx N(\hat{x}_j^k, \hat{P}_j^k) \quad (6)$$

- To sample from $p(s_k, \theta|z_{1:k})$, we choose a trial distribution

$$q(s_k, \theta|s_{k-1}^j, x_{k-1}^j, z_{1:k}) = p(s_k|s_{k-1}^j, x_{k-1}^j, \theta, z_k)p(\theta|s_{k-1}^j, x_{k-1}^j, z_{k-1}) \quad (7)$$

The corresponding importance weight can be calculated as

$$w_j^k \propto w_{j-1}^k p(z_k|s_{k-1}^j, x_{k-1}^j, \theta) \quad (8)$$
RBPF-PL (2/2)

(1) Trial distribution for $s_k$

$$P(s_k|s_{k-1}^j, x_{k-1}^j, \theta, z_k) \approx \frac{\prod_{i=1}^{M} p(z_{i,k}|\hat{x}_{k-1}^j, s_{i,k}, \theta_i)P(s_{i,k}|s_{i,k-1}^j)}{p(z_k|s_{k-1}^j, \hat{x}_{k-1}^j)}$$

(2) Infer the parameter $\theta$, 

- set conjugate prior distribution

$$p(\theta|s_{k-1}^j, x_{k-1}^j, z_{1:k-1}) = N - \text{Inv} - \chi^2(\tilde{\mu}_{k-1}^j, \tilde{\kappa}_{k-1}^j, \tilde{\nu}_{k-1}^j, \tilde{\eta}_{k-1}^j)$$

- update

$$p(\theta|x_k^j, s_k^j, z_{1:k}) \propto p(z_k|\theta, x_k^j, s_k^j)p(\theta|x_{k-1}^j, s_{k-1}^j, z_{1:k-1})$$

$$= N - \text{Inv} - \chi^2(\tilde{\mu}_k^j, \tilde{\kappa}_k^j, \tilde{\nu}_k^j, \tilde{\eta}_k^j)$$

where

$$\{\tilde{\mu}_k^j, \tilde{\kappa}_k^j, \tilde{\nu}_k^j, \tilde{\eta}_k^j\} = f(\tilde{\mu}_{k-1}^j, \tilde{\kappa}_{k-1}^j, \tilde{\nu}_{k-1}^j, \tilde{\eta}_{k-1}^j, z_k, h(x_k^j), s_k^j)$$

Summery

$$p(x_k, s_k, \theta|z_{1:k}) : s_k^j\text{ (optimal)} \rightarrow x_k^j\text{ (EKF)} \rightarrow w_k^j \rightarrow \theta^j \text{ (conjugate prior + sufficient statistics)}$$
Simulation results (1/4)

Simulation parameters

- 3 base stations
- compare 4 algorithms
  1. RBPF using 10 particles: \( \theta \) known, \( p(x_k, s_k|z_{1:k}) \)
     \( s_k \) (optimal) \( \rightarrow \) \( x_k \) (EKF) \( \rightarrow \) \( w^j_k \)
  2. PF-PL using 1000 particles: \( p(x_k, s_k, \theta|z_{1:k}) \)
     \{\( x^j_k, s^j_k \)\} (transition prior) \( \rightarrow \) \( w^j_k \) \( \rightarrow \) \( \theta^j \)
  3. RBPF-PL using 10 particles: \( p(x_k, s_k, \theta|z_{1:k}) \)
     \( s^j_k \) (optimal) \( \rightarrow \) \( x^j_k \) (EKF) \( \rightarrow \) \( w^j_k \) \( \rightarrow \) \( \theta^j \)
  4. RBPF-PL using 10 particles with \( s_k \) known: \( p(x_k, \theta|z_{1:k}) \)
     \( x_k \) (EKF) \( \rightarrow \) \( w^j_k \) \( \rightarrow \) \( \theta^j \)

- LOS: \( N(0, (150m)^2) \); NLOS: \( N(513m, (409m)^2) \)
- hyperparameter setting: \{\( \tilde{\mu}_0 = 1000, \tilde{\kappa}_0 = 1, \tilde{\nu}_0 = 1, \tilde{\eta}_0 = (5\sigma_n)^2 \}\), ‘vague’ prior information on \( \theta \).
- Monte Carlo realizations: \( n_{MC} = 20 \)
Simulation results (2/4)

CDF

Root Square error (m)

RSE $\Delta = \| x - \hat{x} \|

- RBPF(10)
- PF–PL(1000)
- RBPF–PL(10)
- RBPF–PL(sk known, 10)
Simulation results (3/4)

RMSE(k) \triangleq \sqrt{\frac{1}{n_{mc}} \sum_{m=1}^{n_{mc}} \| x - \hat{x} \|^2}

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Simulation results (4/4)

one realization of parameter learning

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Conclusion

- Mobile tracking in mixed LOS/NLOS conditions
  the statistical parameter of NLOS is unknown

- RBPF-PL method
  - estimates the sight condition using particle filters with optimal trial distribution
  - analytically computes mobile state by EKF method
  - updates the posterior of the unknown parameter according to the measurement and
    the estimation on the sight conditions and mobile state.

- Simulation results show the effectiveness
  - 10 particles
  - good tracking performance
  - effective inference on NLOS para.