Bayesian Receiver Autonomous Integrity Monitoring Technique

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“no model is right, but some models are less wrong than others”
Outline

Motivation
Traditional RAIM
Bayesian model comparison
Failure models
Bayesian Receiver Autonomous Integrity Monitoring Method
Simulations
GPS-data test
Conclusions
Motivation

Practically all GNSS-receivers implement RAIM in some form
Detect & exclude faulty observations
Decide if position estimate is reliable

Almost all RAIM methods are based on statistical hypothesis tests
Criticized for convoluted approach

Alternative approach is Bayesian model comparison
Traditional RAIM/FDE

Classic RAIM/FDE is based on frequentist hypothesis testing
Seek to reject hypothesis based on how unlikely it is

RAIM/FDE as a two-stage procedure:

global test

local test

Bayesian RAIM/Pesonen
Frequentist hypothesis testing

Choose test statistic
Choose significance level
Find critical region
Type I and II errors
Assume we have correct models
Test statistic realizes in critical region
Two possible reasons:
1) \( H_0 \) wrong
2) sample is rare
Assume sample is not rare \( \Rightarrow H_0 \) is wrong

“reject the null hypothesis with significance level \( \alpha \)”
Hypotheses (models) can be compared directly using Bayesian theory.

Probability of a model given data

\[ P(M_i | D) = \frac{P(D | M_i) P(M_i)}{P(D)} \]

“probability that the null hypothesis is true is \( P \)”

Sometimes it is easier to think in terms of odds

“What are the odds that the data is from one model?”
Bayesian model comparison

The posterior odds

\[ O_{ij} = \frac{\Pr(M_i \mid D)}{\Pr(M_j \mid D)} = \frac{\Pr(M_i)}{\Pr(M_j)} \times \frac{\Pr(D \mid M_i)}{\Pr(D \mid M_j)} \]

are the prior odds multiplied by the Bayes factor. Bayes factor is the ratio of evidences of models given the data.

Decisions can be made based on the odds

<table>
<thead>
<tr>
<th>( O_{ij} )</th>
<th>( \log_{10} O_{ij} )</th>
<th>Probability for ( M_i ) against ( M_j )</th>
</tr>
</thead>
<tbody>
<tr>
<td>[1, 3.2)</td>
<td>[0, 0.5)</td>
<td>Not worth more than barely a mention</td>
</tr>
<tr>
<td>[3.2, 10)</td>
<td>[0.5, 1)</td>
<td>Substantial</td>
</tr>
<tr>
<td>[10, 31.6)</td>
<td>[1, 1.5)</td>
<td>Strong</td>
</tr>
<tr>
<td>[31.6, 100)</td>
<td>[1.5, 2)</td>
<td>Very strong</td>
</tr>
<tr>
<td>[100, ( \infty ))</td>
<td>[2, ( \infty ))</td>
<td>Decisive</td>
</tr>
</tbody>
</table>
RAIM technique employing Bayesian model comparison theory

Build statistical models for the observations
(at most one failure at a time)

\[ M_0 : y = H_0 x_0 + v \]

\[ M_i : y = H_0 x_0 + b_i e_i + v = \begin{bmatrix} H_0, e_i \end{bmatrix} \begin{bmatrix} x_0 \\ b_i \\ x_i \end{bmatrix} + v, \; i = 1, \ldots, n, \]

Prior distribution for the bias (in addition to state)
The Bayes factor

$$B_{ij} = \frac{c_i}{c_j} \times \exp(g_i(z_i) - g_j(z_j))$$

$$c_i = \begin{cases} 
1, & i = 0 \\
\frac{1}{\sqrt{\sigma_b^2(e_i^T S^{-1} e_i + \frac{1}{\sigma_b^2})}}, & i \neq 0
\end{cases}$$

$$g_i(z_i) = \frac{1}{2} \begin{cases} 
0, & i = 0 \\
\frac{(y - H_i \mu_i)^T S^{-1} e_i^2}{e_i^T S^{-1} e_i + \frac{1}{\sigma_b^2}}, & i \neq 0
\end{cases}$$

$$z_i = y - H_i \mu_i$$

$$S = HP_{x_0}H^T + R$$

Prior

$$P_{i0} = \frac{\mathbb{P}(n - 1 \text{ channels are clean and } 1 \text{ is contaminated})}{\mathbb{P}(n \text{ channels are clean})} = \frac{\mathbb{P}(\text{channel is clean})^{n-1} \mathbb{P}(\text{channel is contaminated})}{\mathbb{P}(\text{channel is clean})^n} = \frac{(1 - \epsilon)^{n-1} \epsilon}{(1 - \epsilon)^n} = \frac{\epsilon}{1 - \epsilon}$$

(16)
Position solution is the posterior distribution given the most plausible model

Most probable model is the model with the best odds against the ‘null’ model

$$k = \arg \max_i O_{i0}$$

Position solution

$$p(x_k|y, M_k)$$  Use as a prior at next time step

From posterior we can compute measures of quality
How probable that the error is at most \(r\)?
Bayesian Receiver Autonomous Integrity Monitoring Method

Best odds against the ‘null’ model

Condition for correct biased observation to be identified

Check if the odds for the best model are good enough
We compare the detection performance of RAIM/FDE and BRAIM

Observation noise $N(0,10^2)$

Every twenty epochs, one channel generates blunder observations from $N(0,\sigma_c^2)$

Satellites generated uniformly on $[-10^5,10^5] \times [-10^5,10^5] \times [10^5,10^5+10^2]$.

Tracks simulated using constant velocity model.
## Simulations (large variance priors)

<table>
<thead>
<tr>
<th>Test</th>
<th>( n )</th>
<th>( \sigma_e^2 )</th>
<th>System OK (%)</th>
<th>Warning (%)</th>
<th>Failure (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A_1,A_2</td>
<td>5</td>
<td>100^2,200^2</td>
<td>Correct decision</td>
<td>69</td>
<td>31</td>
</tr>
<tr>
<td>B_1,B_2</td>
<td>6</td>
<td>100^2,200^2</td>
<td>Wrong decision</td>
<td>33</td>
<td>67</td>
</tr>
<tr>
<td>C_1,C_2</td>
<td>7</td>
<td>100^2,200^2</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 4. Test \( C_1 \) results with \( T=10, \ \sigma_b^2=80^2, \ \varepsilon=0.6 \) (large-variance prior for parameters).

![Graph showing test results](image)
Simulations (small variance priors)

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</thead>
<tbody>
<tr>
<td>A_1,A_2</td>
<td>5</td>
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<td>Correct decision</td>
<td>74</td>
<td>26</td>
</tr>
<tr>
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<td>100^2,200^2</td>
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<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 3. Test \( C_1 \) results with \( T=10, \sigma_e^2=80^2, \epsilon=0.6 \) (small-variance prior for parameters).
GPS-data test

BRAIM was tested using GPS-observations
Artificial test was carried out by dropping good quality observations and by using the observation with the worst CNR

Bayesian RAIM/Pesonen
Bayesian model comparison can be used as an alternative to traditional hypothesis test based-RAIM methods

Frequentists:
“reject the null hypothesis with significance level $\alpha$”

Bayesians:
“probability that the null hypothesis is true is $P$”

BRAIM has
Low computational complexity (under certain conditions)
No restrictions on number of faulty observations (models)
Natural expansion to time series
Position domain investigations needed
Thank you!
Questions?

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