Gaussian Mixture Filters and Hybrid Positioning

ION GNSS 2007
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Outline

- Bayesian filtering
- (Problem) Extended Kalman Filter
- (Solution) Splitting prior into a GM
- Suburban results
- Real data example
- Conclusion
Bayesian filtering
Bayesian filtering

Initial state: \( x_0 \)

Motion model: \( x_{k+1} = f(x_k) + w_k \)

Measurement model: \( y_k = h(x_k) + v_k \)

\[
p(x_k|y_{1:k}) = \frac{p(y_k|x_k)p(x_k|y_{1:k-1})}{\int p(y_k|x_k)p(x_k|y_{1:k-1}) \, dx_k}
\]

current meas.  state model and past meas.

normalization
Example: One range meas. and Gaussian prior
Extended Kalman Filter (EKF)
EKF likelihood: linearize measurement
EKF posterior vs. correct posterior
EKF does not work properly

Mahalanobis distance

Lissack–Fu distance

Distance between prior mean and base station
Splitting prior into a Gaussian Mixture
Splitting prior into a Gaussian Mixture
Posterior is also Gaussian Mixture
EKF and GMF posteriors vs. correct posterior
GMF works better than EKF

Mahalanobis distance

Lissack–Fu distance

Distance between prior mean and base station
When should we split prior?
When should we split prior? If nonlinearity is $> 0$
Suburban results
# Suburban results

<table>
<thead>
<tr>
<th>Solver</th>
<th>Time</th>
<th>Err.</th>
<th>Err.</th>
<th>Inc.</th>
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<tbody>
<tr>
<td>EKF</td>
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<tr>
<td>UKF</td>
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<td>183</td>
<td>361</td>
<td>1.5</td>
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<td>GMF&lt;sub&gt;ENC2007&lt;/sub&gt;</td>
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GMF<sub>ENC2007</sub> approximate likelihood as GM.
Suburban results with Cell ID

<table>
<thead>
<tr>
<th></th>
<th>Time</th>
<th>Err.</th>
<th>Err.</th>
<th>Inc.</th>
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<td>%</td>
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</table>

Diagram: Base station 500 m
Real data example
GPS + simulated BS range meas.

Start/Finish

- True
- Building
- GMF
- EKF
- PF

5000 – p.21/24
Conclusion

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