A Modified Kalman Filter for Hybrid Positioning

ION GNSS 2006
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Outline

- Motivation
- Problem
- Mathematical background
- Example
- Some results
- Conclusion
Not enough measurements for unique solution...
...restrictive information is necessary.
Combine restrictive information and Kalman-type filters?

Restrictive information tells that state is inside some area. For example sector information.

Kalman-type filters are linear filters which approximate only first two moments of distribution.

\[
\hat{x}^+ = \hat{x}^- + K(y - \hat{y})
\]

\[
P^+ = P^- - KP_yK^T
\]

For example: Extended Kalman Filter (EKF) and Unscented Kalman Filter (UKF).
K-type filter approximates Bayes filter

Initial state: \( x_0 \)
Motion model: \( x_{k+1} = f(x_k) + w_k \)
Measurement model: \( y_k = h(x_k) + v_k \)

\[
p(x_k | y_{1:k}) = \frac{\int p(y_k | x_k) p(x_k | y_{1:k-1}) \, dx_k}{\int p(y_k | x_k) p(x_k | y_{1:k-1}) \, dx_k}
\]

K-type filters \( x_k | y_{1:k} \approx N(\hat{x}_k^+, P_k^+) \).
Two stage computation of $p(x_k|y_{1:k})$ mean and covariance.

\[
p(y_k|x_k) = p(y_{\text{res}}^k|x_k)p(y_{\text{other}}^k|x_k),
\]

\[
p(x_k|y_{1:k}) \propto p(y_{\text{res}}^k|x_k)p(x_k|y_{\text{other}}^k, y_{1:k-1}),
\]

- **Restrictive info:** $p(y_{\text{res}}^k|x_k) = \chi_A(x_k)$.
- **Other measurements:**
  \[
p(x_k|y_{\text{other}}^k, y_{1:k-1})^{\text{K-type}} \approx \mathcal{N}(\mu_{\text{old}}, \Sigma_{\text{old}}).
\]
Approximate restrictive area

\[ p(y_k|x_k) = \chi_A(x_k)p(y_k^{\text{other}}|x_k) \approx \chi_{A'}(x_k)p(y_k^{\text{other}}|x_k), \]

where \( A \subset A' \),

\( A' = \{ x | \| Ax - Ax_{\text{mid}} \| \leq \alpha \} \)

and \( A\Sigma_{\text{old}}A^T = I \).

We can then compute mean and covariance.
New mean and covariance

\[ \mu_{\text{new}} = \mu_{\text{old}} + \sum_{\text{old}} A^T \epsilon \]

\[ \Sigma_{\text{new}} = \Sigma_{\text{old}} - \sum_{\text{old}} A^T \Lambda A \Sigma_{\text{old}}, \]

We approximate the posterior by a Gaussian.
Simulation variances:

- Base station range meas.: $80^2 \text{ m}^2$
- Pseudorange meas.: $20^2 \text{ m}^2$
- Delta pseudorange meas.: $\frac{2^2 \text{ m}^2}{\text{s}^2}$
0-2 base stations: BOX-solver is better than EKF

100 simulations:

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<th>Solver</th>
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<th>Err.</th>
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<th>Err.</th>
<th>&lt;50m</th>
<th>Inc.</th>
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<td>μ</td>
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<td>%</td>
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<td>436</td>
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* uses only restrictive info.
Suburban: EKFBOX is almost as accurate as SMC

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Conclusion

The new algorithm

- Uses restrictive information
- Gives better results (accuracy, consistency)
- Is fast to compute
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Thank you for your attention!
Questions?

http://math.tut.fi/posgroup/