Hybrid positioning algorithms

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The umbrella term of my research area is *Bayesian filtering in Hybrid Positioning*. Very briefly, the research problem is to find a good filter and develop it even better to hybrid positioning when optimal criterions are accuracy, consistency, robustness and low complexity. In this report I explain briefly what is hybrid positioning, Bayesian filtering and optimal criterions for filter evaluations. After that I tell pros and cons of some existing filters and finally I list my current research topic on this area.

Hybrid positioning means that measurements used in positioning come from many different sources e.g. Global Navigation Satellite System (GNSS), Inertial Measurement Unit (IMU), or local wireless networks such as a cellular network, WLAN, or Bluetooth. Range, pseudorange, deltarange, altitude, and compass measurements are examples of typical measurements in hybrid positioning. In hybrid positioning, there are lot of other measurements such as restrictive information, which can be used in positioning. Restrictive information tells that state e.g. position, is inside some area. In the simplest case, the area is a half-space. For example, base station sector and maximum range information are restrictive information.

Bayesian filter and its approximations are very commonly used in positioning and Bayesian filter framework suit very well in hybrid positioning. Because of this we concentrate our study in Bayesian framework. Bayesian filtering problem formulation includes three things: initial state \( x_0 \), state model and measurement model. State model tells how the next state \( x_{k+1} \) depend on current state \( x_k \),

\[
x_{k+1} = f(x_k) + w_k.
\]

(1)

Measurement model tells how measurements depend on current state

\[
y_k = h(x_k) + v_k.
\]

(2)

Here error terms \( w_k \) and \( v_k \) are random variables. The aim of Bayesian filtering is to solve state conditional probability density function (cpdf)

\[
p(x_k | y_{1:k} = y_{1:k}),
\]

(3)

when conditional \( y_{1:k} \triangleq \{y_1, \ldots, y_k\} \) are past and current measurements. In hybrid positions case the cpdf cannot be determined analytically. Because of this there are many approximative solutions for example Extended Kalman Filter (EKF), Second Order Extended Kalman Filter (EKF2), Position Kalman Filter (PKF), Gaussian Mixture Filter (GMF), Particle Filter (PF), Grid Based Method (GBM) and many other
approximative solutions. The aim of study is to apply and modify these solutions to hybrid positioning. This includes not only how to solve cpdf efficiently but also how to model the problem in Bayesian framework. Of course all solutions have their own pros and cons, that we explain later. When we compare different filters with each other, it is important to know what is good filter. The features of a good filter in positioning are following.

**Accuracy** means that state estimates are accurate enough for personal positioning. Naturally accuracy depends mainly on the number of measurement but it also depends on how faithfully we can solve cpdf. It is also interesting thing to study what is sufficient number of measurement for certain accuracy.

**Consistency** briefly means that filter works correctly so that calculated cpdf corresponds to actual cpdf. The necessary condition of that is that state estimates and covariance matrix of state estimates corresponds to each other. Actually this is almost sufficient condition because almost only way fulfill this condition in time series is keep in mind what is actual cpdf.

**Robustness** means that filter tolerates some errors in measurements and dynamic model. Of course this is modeling question, how we model state dynamic and measurement equation so that they tolerate some modeling errors.

**Low complexity** means that solution can be implement in a portable terminal.

PKF, EKF and EKF2 are Kalman type filters which means that filter approximates distribution of the state as a Gaussian. As a rule of thumb we can say that Kalman type filters are fast to compute but they are not optimal solutions, whereas PF and GBM need more computation but are almost optimal solutions of problem. Big problem about EKF and EKF2 is inconsistency, problem exist especially when true density function of state is multimodal. Example of situation where EKF is inconsistent and EKF2 and PKF are consistent is in Figure 1. Figure also shows how PKF is not optimal for underdetermined cases. Here we list some basic features, pros and cons of existing hybrid positioning filters.

**PKF:** The Position Kalman Filter works by filtering a sequence of static position and velocity solutions. Because of this PKF is almost always consistent but especially in underdetermined case it is not optimal solutions.

**EKF:** The Extended Kalman Filter solves the filtering problem by linearizing the measurement function. In hybrid position framework EKF is sometimes inconsistent this means that error estimates do not correspond to true errors (Figure 1).

**EKF2:** The Second Order Extended Kalman Filter is an elaboration of EKF that takes into consideration the nonlinearity of the measurement models. EKF2 is not so often inconsistent than EKF but there are some cases where also EKF2 is inconsistent.
GMF: Gaussian Mixture Filter. Filter is GMF if approximation of distribution of state is mixture of Gaussians. So GMF is in fact family of filters, also PKF, EKF and EKF2 are GMFs. It is also good to notice that we can approximate every density function arbitrary well with a mixture of Gaussians.

PF: Particle Filter based on Monte Carlo integration and it approximates density function with random points. It is shown that if number of points increase, the state approximation approaches to the correct state distribution. Unfortunately then also computation increases very much.

GBM: Grid Based Method is basically almost same than PF expect that GBM points are not random but explicitly defined.

As we have seen this research area is quite large. Because of that I have concentrated my study in some specific topics: below.

GMF When we have for example two base station range measurements, altitude measurement and uniform prior then commonly posterior distribution is bimodal. So then it is reasonable to use GMF with two mixture components. Now I study how we can use GMF with small number of mixture components in hybrid positioning.

Restrictive information Restrictive information, which tells that state is inside some area, is easy to use sophisticatedly in PF and GBM. But in GMF it is not straightforward to use restrictive information.
**Robustifying filters** One major topic of my research is how to robustify filters, especially GMF, in hybrid positioning application. There are some popular ways to robustify filters if we assume that prior information is true. But because of inconsistency problem, we cannot always assume that our prior information is correct. So we cannot robustify our filter with assumption that prior is correct, because this assumption, when prior is wrong, can cause even bigger problem than without robustifying.

**Performance test** It is not obvious how to evaluate filters so that the test is fair and possible to compute. Because of this we must to study and develop performance test to filters evaluations. I have especially studied how to test consistency without the Gaussian assumptions.