ON THE POSET INDUCED BY THE SIMPLE MINOR RELATION AND ITS JOIN-IRREDUCIBLE MEMBERS

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In this presentation we shall be interested in a quasi-ordering of Boolean functions which is know as the simple minor relation. This quasi-order can be described as follows: a function g is said to be *simple minor* of a function f, denoted $g \leq f$, if g can be obtained from f by identification of variables, permutation of variables or addition of dummy variables.

The importance of the simple minor relation $\leq$ in Boolean function definability was made apparent by Ekin, Foldes, Hammer and Hellerstein who showed that the classes (or properties) of Boolean functions definable by functional equations coincide exactly with the initial segments of this quasi-order. This correspondence to function class definability led to several studies of the simple minor relation. As any quasi-order, the simple minor relation $\leq$ on the set Ω of all Boolean functions induces a partial order on the set $\tilde{\Omega}$ made of equivalence classes, where properties of $\leq$ are easier to express. Several results concerning this poset have been established and interesting connections to hypergraph theory have been recently revealed.

In this talk we will survey these and other results concerning the poset $\tilde{\Omega}$. We start by presenting some properties of this poset and establish connections to the equational approach to function class definability. Then we will present a classification of $\tilde{\Omega}$ (showing that it has a sort of universal property among countable posets) and address some open problems. In particular, we will consider the question of determining the join-irreducible elements of this poset, i.e., elements having a unique lower cover in $\tilde{\Omega}$. Using a complete correspondence between Boolean functions and hypergraphs, we will see that join-irreducibility translates into combinatorial properties of hypergraphs such as set transitivity and monomorphicity. As particular cases, we will consider Steiner systems and graphs and present characterizations of those corresponding to join-irreducible members of $\tilde{\Omega}$. We will discuss some results obtained jointly with M. Bouaziz, S. Foldes, E. Lehtonen and M. Pouzet.