

# HYPERHOLOMORPHIC FUNCTIONS IN A QUATERNIONIC AND FUETER OPERATOR FRAMEWORK

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## ABSTRACT

We start considering hypermonogenic functions in the Clifford algebra  $\mathcal{Cl}_{0,3}$ . We give a short proof of a main property; that the function  $x \rightarrow x^n$  is hypermonogenic. Using the idea behind this proof we then give sufficient conditions for a Cullen-regular quaternionic function in the sense of Gentili-Struppa in order to satisfy Fueter's Theorem. Using this conditions we introduce and motivate a class of Cullen-regular functions that we call *quaternionic hyperholomorphic*. In particular the case studied by G. Gentili and D.C Struppa, when the Cullen-regular function is defined on a ball centered in the origin of radius  $R$  is shown to be hyperholomorphic. We then discuss other properties of this class of quaternionic functions and how it compares to the class of hypermonogenic functions in  $\mathcal{Cl}_{0,3}$ .

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