An Inverse Boundary Value Problem for Dirac Operators with Right-Multiplying Potentials

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We consider inverse boundary value problems for Dirac operators with potentials whose algebraic properties are related to multiplication from the right in a Clifford algebra. An example result is as follows.

Let A be the universal Clifford algebra for $\mathbb{C}^m$, $m \geq 3$. Decompose $A = A_+ \oplus A_-$, where A_+ and A_- are the subspaces generated by the products of even and odd numbers of elements of $\mathbb{C}^m$, respectively.

Let $\Omega \subset \mathbb{R}^m$ be an open and bounded set with smooth boundary, and consider the boundary value problem

$$\begin{cases} Du + u\,v(x) & = 0 \qquad \text{in } \Omega \\ \quad u_+|_{\partial\Omega} & = f \qquad \text{on } \partial\Omega \end{cases} .$$

Here D is the standard Dirac operator, the potential v is an A-valued function that acts on u by multiplication from the right, and $u = u_+ + u_-$, where $u_\pm$ are $A_\pm$-valued. If the problem is well-posed, the boundary measurement is the mapping $f \mapsto u_-|_{\partial\Omega}$.

We show that if the potentials $v_1, v_2 \in W^{1,\infty}(\Omega; A_+)$ satisfy $v_1|_{\partial\Omega} = v_2|_{\partial\Omega}$ and their boundary measurements coincide, then $v_1 = v_2$ in Ω.